Cambridge International AS & A Level
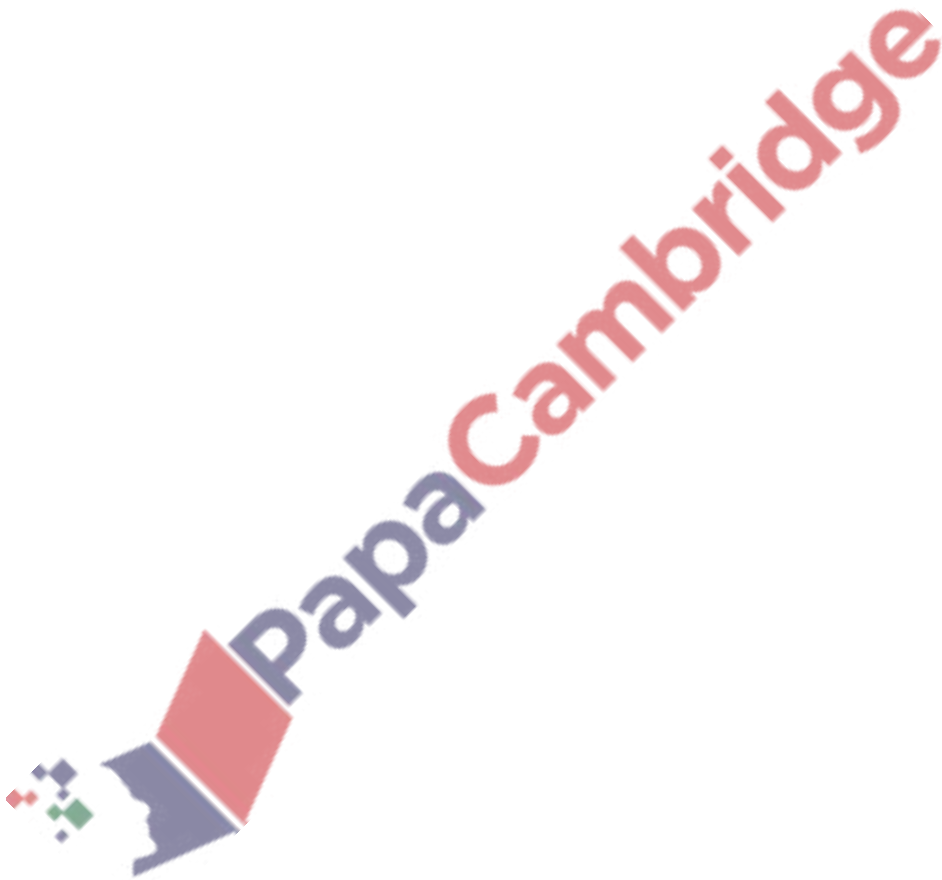
MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

Chapter 2

Logarithmic and exponential functions

42. 9709_s20_qp_21 Q: 1

Solve the equation

$$\ln(x + 1) - \ln x = 2 \ln 2.$$

[3]

43. 9709_s20_qp_22 Q: 1

Given that $2^y = 9^{3x}$, use logarithms to show that $y = kx$ and find the value of k correct to 3 significant figures.   [3]

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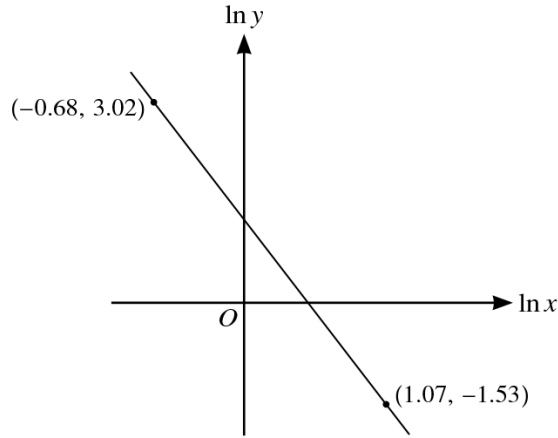
44. 9709_s20_qp_22 Q: 4

The variables x and y satisfy the equation $y = Ax^{-2p}$, where A and p are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(-0.68, 3.02)$ and $(1.07, -1.53)$, as shown in the diagram.

Find the values of A and p.   [5]

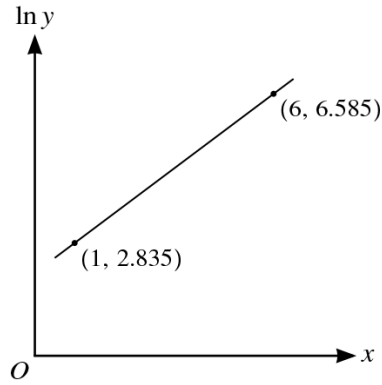..

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45. 9709_w20_qp_22 Q: 2

Given that $\dfrac{2^{3x+2} + 8}{2^{3x} - 7} = 5$, find the value of 2^{3x} and hence, using logarithms, find the value of x correct to 4 significant figures. [5]

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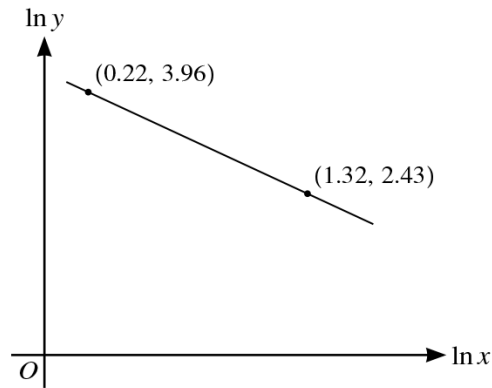
46. 9709_m19_qp_22 Q: 3

The variables x and y satisfy the equation $y = Ae^{px+p}$, where A and p are constants. The graph of $\ln y$ against x is a straight line passing through the points $(1, 2.835)$ and $(6, 6.585)$, as shown in the diagram. Find the values of A and p. [5]

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47. 9709_s19_qp_21 Q: 1

Show that $\ln(x^3 - 4x) - \ln(x^2 - 2x) \equiv \ln(x + 2)$. [3]

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48. 9709_w19_qp_22 Q: 2

(i) Solve the equation $|4x + 5| = |x - 7|$. [3]

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(ii) Hence, using logarithms, solve the equation $|2^{y+2} + 5| = |2^y - 7|$, giving the answer correct to 3 significant figures. [2]

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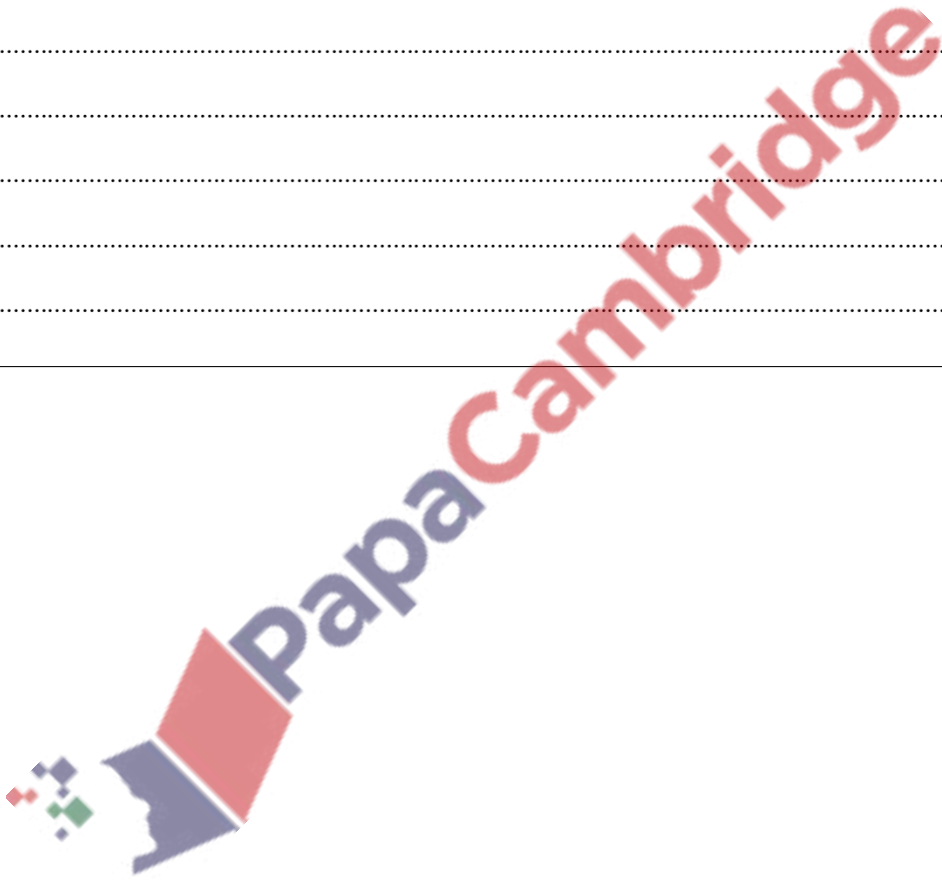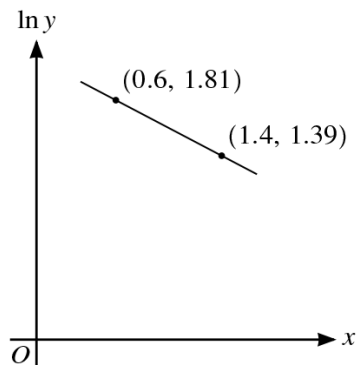
49. 9709_w19_qp_22 Q: 3

The variables x and y satisfy the equation $y = kx^a$, where k and a are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(0.22, 3.96)$ and $(1.32, 2.43)$, as shown in the diagram. Find the values of k and a correct to 3 significant figures. [5]

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50. 9709_s18_qp_21 Q: 1

Solve the equation $3e^{2x} - 82e^x + 27 = 0$, giving your answers in the form $k \ln 3$.　　　　[5]

51. 9709_s18_qp_21 Q: 2
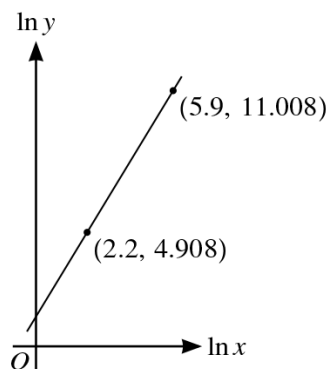
The variables x and y satisfy the equation $y = A \times B^{\ln x}$, where A and B are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(2.2, 4.908)$ and $(5.9, 11.008)$, as shown in the diagram. Find the values of A and B correct to 2 significant figures. [5]

52. 9709_s18_qp_22 Q: 4

(i) Solve the equation $2\ln(2x) - \ln(x+3) = 4\ln 2$. [5]

(ii) Hence solve the equation

$$2\ln(2^{u+1}) - \ln(2^u + 3) = 4\ln 2,$$

giving the value of u correct to 4 significant figures. [2]

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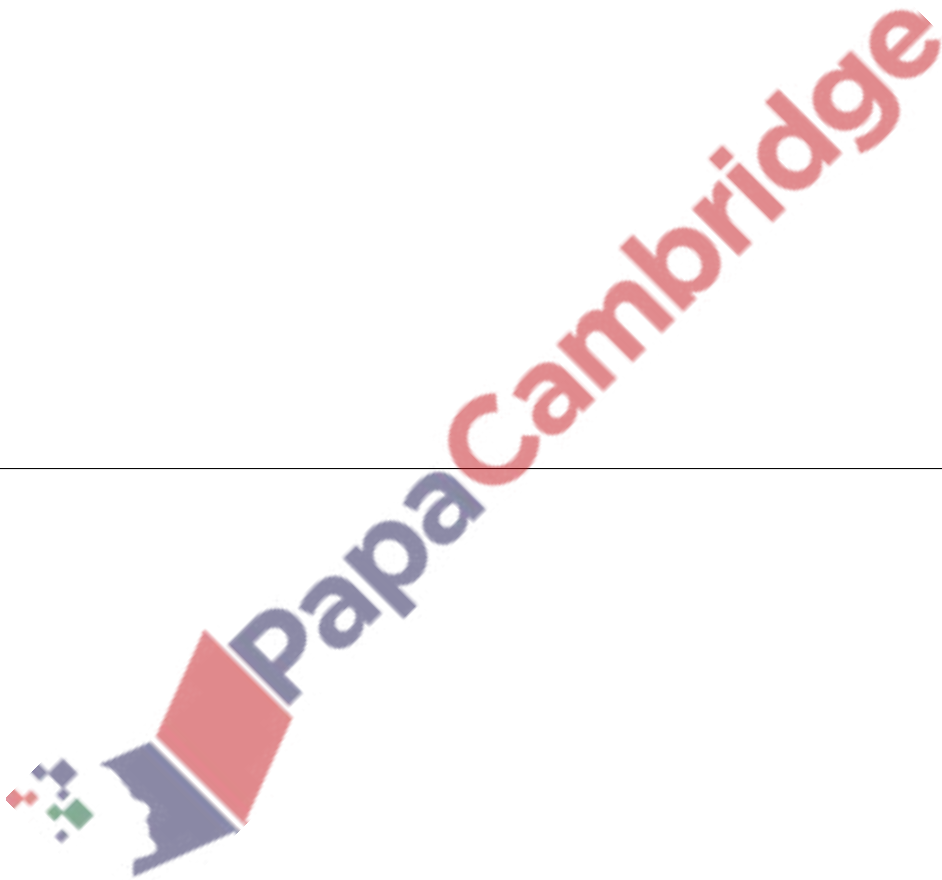
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53. 9709_w18_qp_21 Q: 1

(i) Solve the equation $|9x - 2| = |3x + 2|$.   [3]

(ii) Hence, using logarithms, solve the equation $|3^{y+2} - 2| = |3^{y+1} + 2|$, giving your answer correct to 3 significant figures.   [2]

54. 9709_w18_qp_22   Q: 2

Given that $9^x + 3^x = 240$, find the value of 3^x and hence, using logarithms, find the value of x correct to 4 significant figures. [5]

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55. 9709_m17_qp_22 Q: 1

Solve the equation $2\ln(2x) - \ln(x + 3) = \ln(3x + 5)$.   [5]

56. 9709_m17_qp_22 Q: 3

(i) Solve the inequality $|2x - 5| < |x + 3|$. [4]

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(ii) Hence find the largest integer y satisfying the inequality $|2 \ln y - 5| < |\ln y + 3|$. [2]

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57. 9709_s17_qp_21 Q: 1

Given that $5^x = 3^{4y}$, use logarithms to show that $y = mx$ and find the value of the constant m correct to 3 significant figures.   [3]

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58. 9709_s17_qp_22 Q: 2

Use logarithms to solve the equation $3^{x+4} = 5^{2x}$, giving your answer correct to 3 significant figures.

[4]

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59. 9709_s17_qp_22 Q: 5

The variables x and y satisfy the equation $y = \dfrac{K}{a^{2x}}$, where K and a are constants. The graph of $\ln y$ against x is a straight line passing through the points $(0.6, 1.81)$ and $(1.4, 1.39)$, as shown in the diagram. Find the values of K and a correct to 2 significant figures.   [6]

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60. 9709_w17_qp_21 Q: 1

Solve the equation $\ln(3x + 1) - \ln(x + 2) = 1$, giving your answer in terms of e. [4]

61. 9709_w17_qp_21 Q: 3

It is given that the variable x is such that

$$1.3^{2x} < 80 \qquad \text{and} \qquad |3x - 1| > |3x - 10|.$$

Find the set of possible values of x, giving your answer in the form $a < x < b$ where the constants a and b are correct to 3 significant figures. [7]

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62. 9709_w17_qp_22 Q: 1

Use logarithms to solve the equation $5^{3x-1} = 2^{4x}$, giving your answer correct to 3 significant figures.

[4]

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63. 9709_m16_qp_22 Q: 3

It is given that k is a positive constant. Solve the equation $2\ln x = \ln(3k + x) + \ln(2k - x)$, expressing x in terms of k. [5]

64. 9709_s16_qp_21 Q: 3

Given that $3e^x + 8e^{-x} = 14$, find the possible values of e^x and hence solve the equation $3e^x + 8e^{-x} = 14$ correct to 3 significant figures. [6]
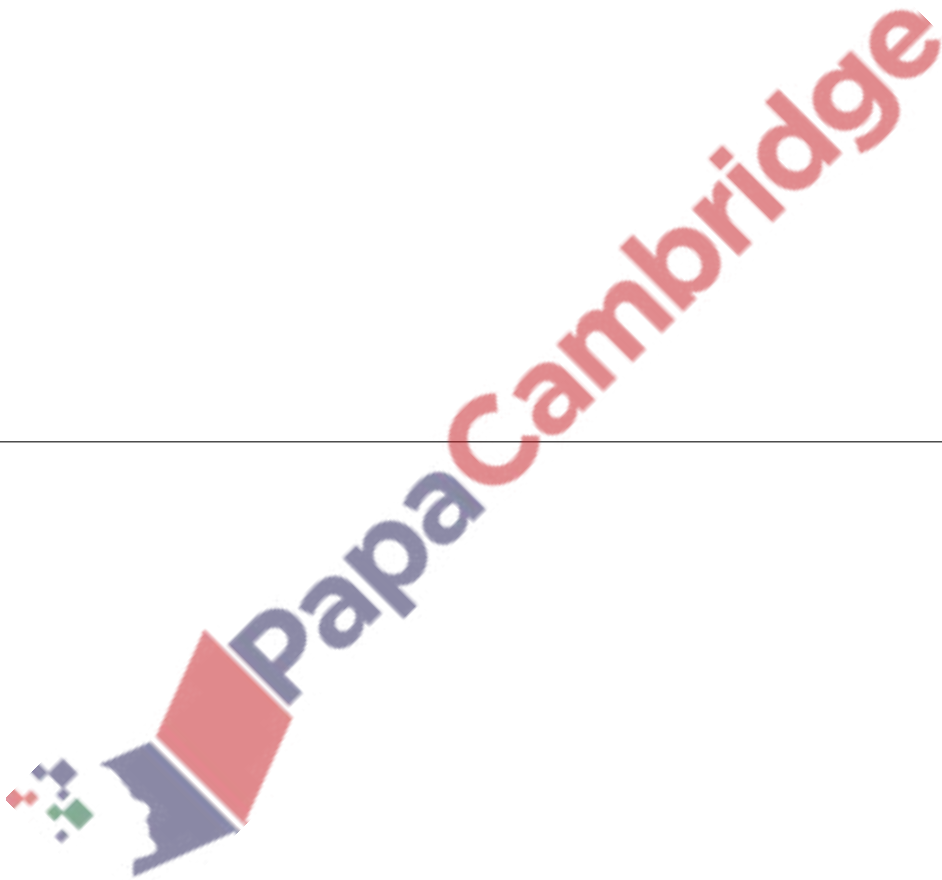
65. 9709_s16_qp_22 Q: 1

Given that $5^{3x} = 7^{4y}$, use logarithms to find the value of $\dfrac{x}{y}$ correct to 4 significant figures.   [3]

66. 9709_w16_qp_21 Q: 1

(i) It is given that x satisfies the equation $3^{2x} = 5(3^x) + 14$. Find the value of 3^x and, using logarithms, find the value of x correct to 3 significant figures. [4]

(ii) Hence state the values of x satisfying the equation $3^{2|x|} = 5(3^{|x|}) + 14$. [1]

67. 9709_w16_qp_21 Q: 2
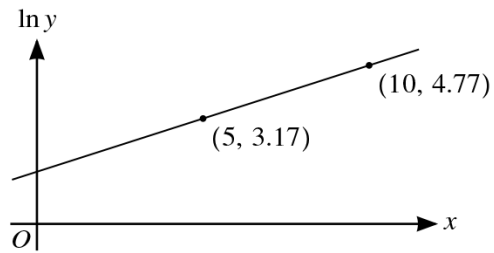
The variables x and y satisfy the equation $y = Ae^{px}$, where A and p are constants. The graph of $\ln y$ against x is a straight line passing through the points $(5, 3.17)$ and $(10, 4.77)$, as shown in the diagram. Find the values of A and p correct to 2 decimal places.   [5]

68. 9709_w16_qp_22 Q: 2

(i) Given that $\dfrac{1 + 4^y}{3 + 2^y} = 5$, find the value of 2^y. [3]

(ii) Use logarithms to find the value of y correct to 3 significant figures. [2]

69. $9709\_w16\_qp\_23$ Q: 2

ln y

(2.11, 4.81)

(1.28, 3.69)

O ln x
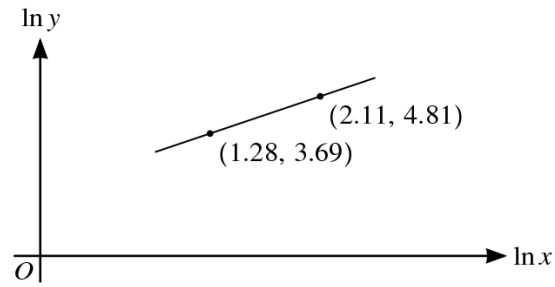
The variables x and y satisfy the equation $y = Kx^p$, where K and p are constants. The graph of ln y against ln x is a straight line passing through the points $(1.28, 3.69)$ and $(2.11, 4.81)$, as shown in the diagram. Find the values of K and p correct to 2 decimal places.   [5]
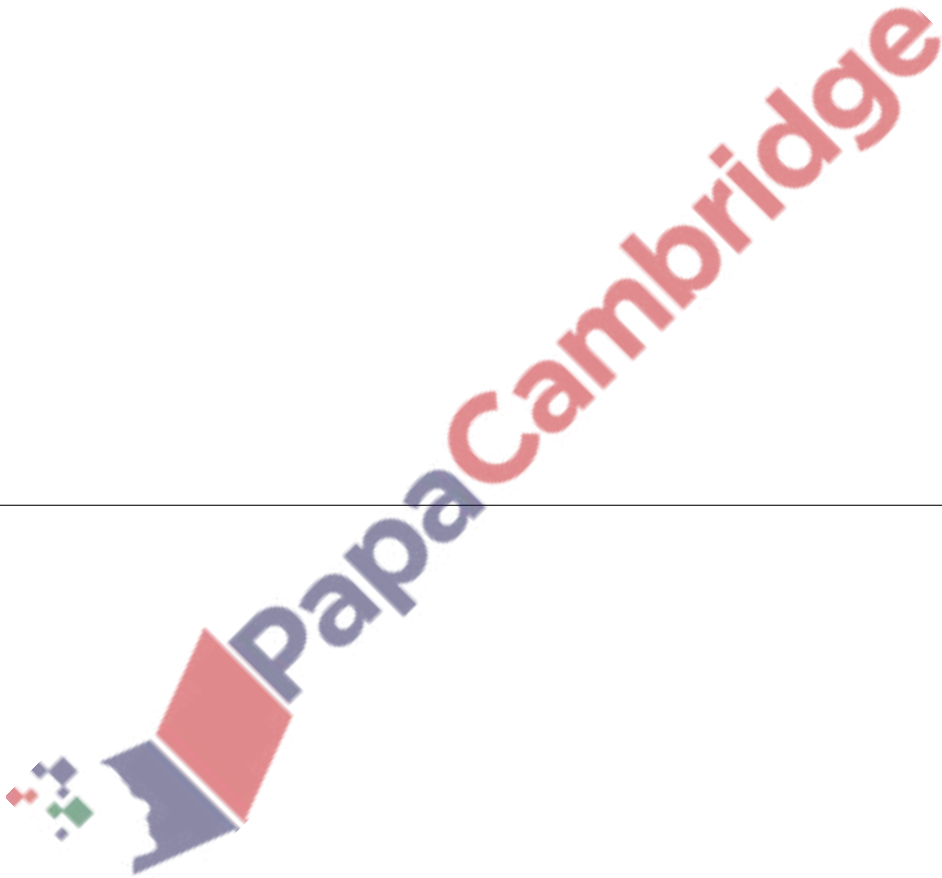
70. 9709_w16_qp_23 Q: 6

A curve has parametric equations

$$x = \ln(t + 1), \quad y = t^2 \ln t.$$

  (i) Find an expression for $\dfrac{dy}{dx}$ in terms of t.   [5]

  (ii) Find the exact value of t at the stationary point.   [2]

  (iii) Find the gradient of the curve at the point where it crosses the x-axis.   [2]
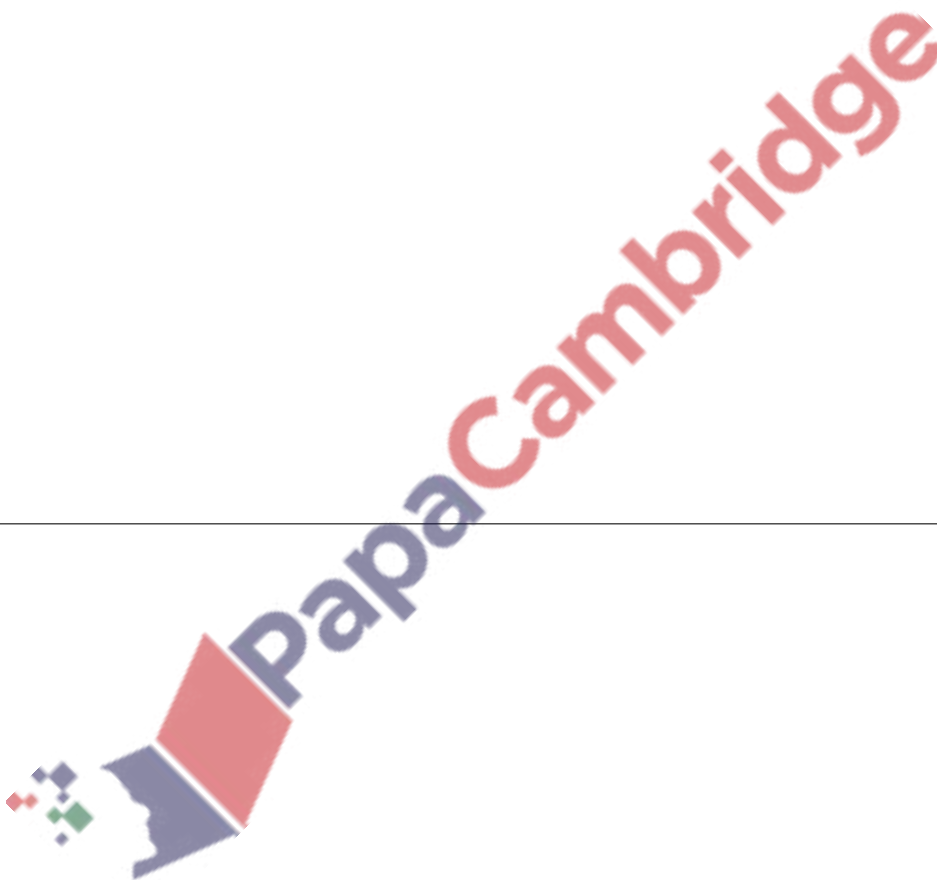
71. 9709_s15_qp_21 Q: 1

(i) Solve the equation $|3x + 4| = |3x - 11|$.   [3]

(ii) Hence, using logarithms, solve the equation $|3 \times 2^y + 4| = |3 \times 2^y - 11|$, giving the answer correct to 3 significant figures.   [2]

72. 9709_s15_qp_21 Q: 2
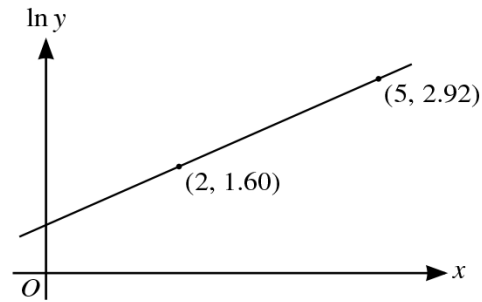
The variables x and y satisfy the equation

$$y = Ae^{p(x-1)},$$

where A and p are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2, 1.60)$ and $(5, 2.92)$, as shown in the diagram. Find the values of A and p correct to 2 significant figures. [5]
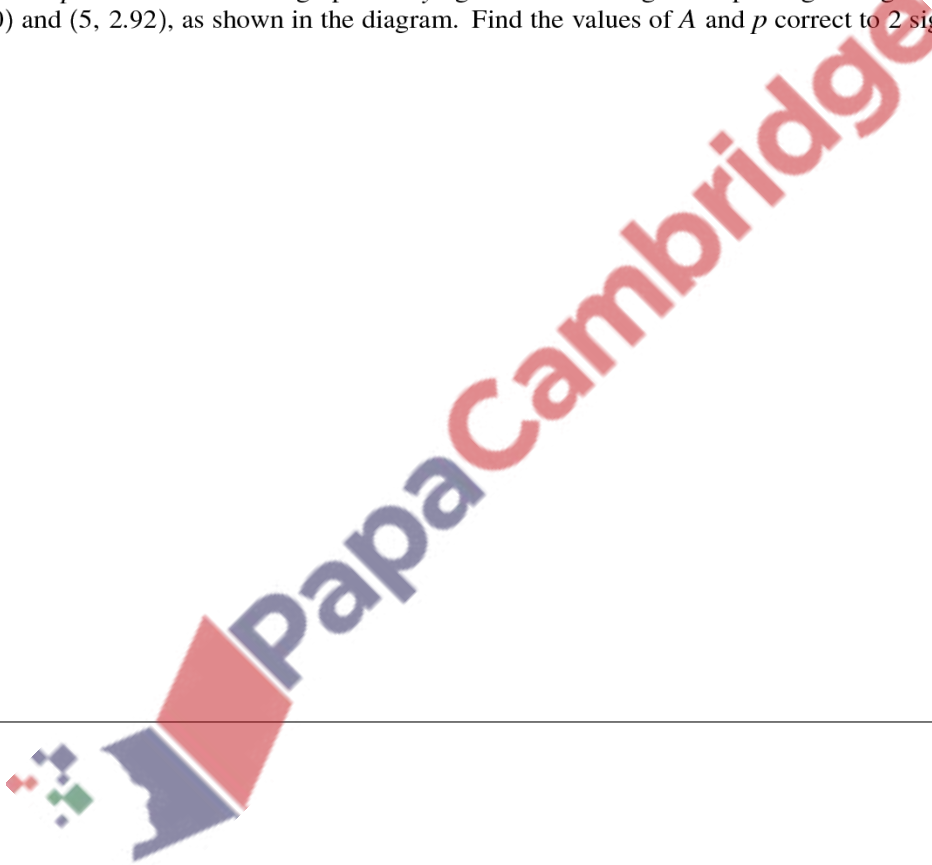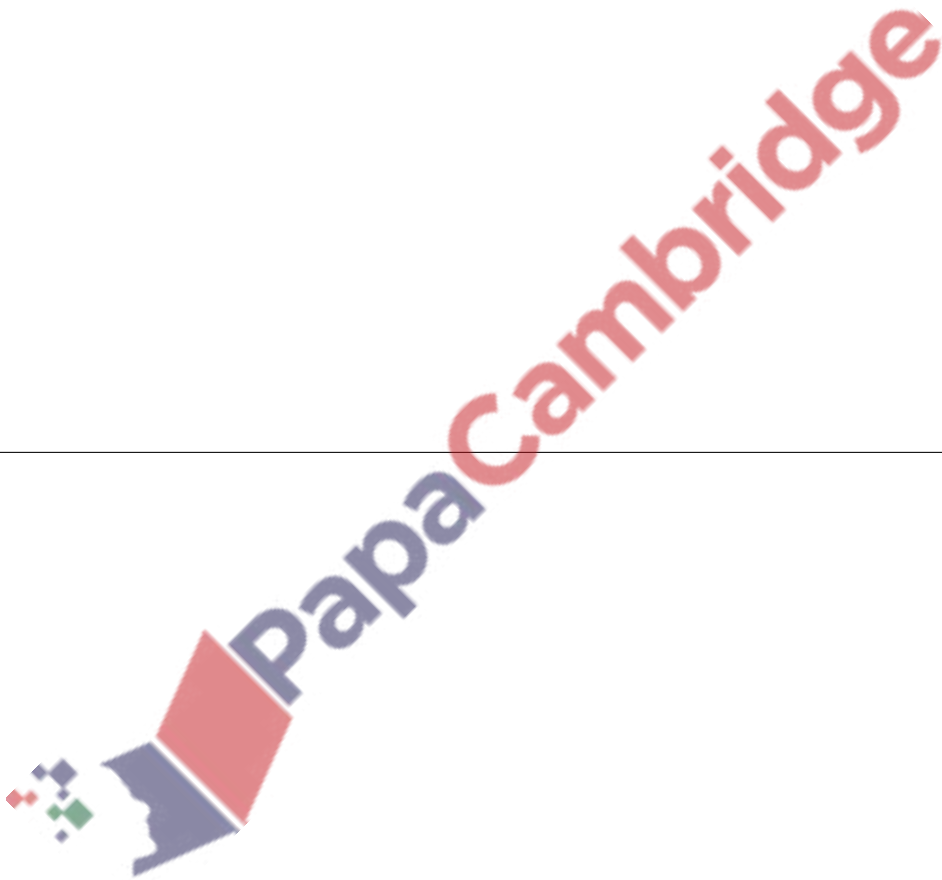
73. 9709_s15_qp_22   Q: 1

(i) Use logarithms to solve the equation $2^x = 20^5$, giving the answer correct to 3 significant figures.

[2]

(ii) Hence determine the number of integers n satisfying

$$20^{-5} < 2^n < 20^5.$$

[2]

74. 9709_s15_qp_22   Q: 7

(a) Find the gradient of the curve

$$3\ln x + 4\ln y + 6xy = 6$$
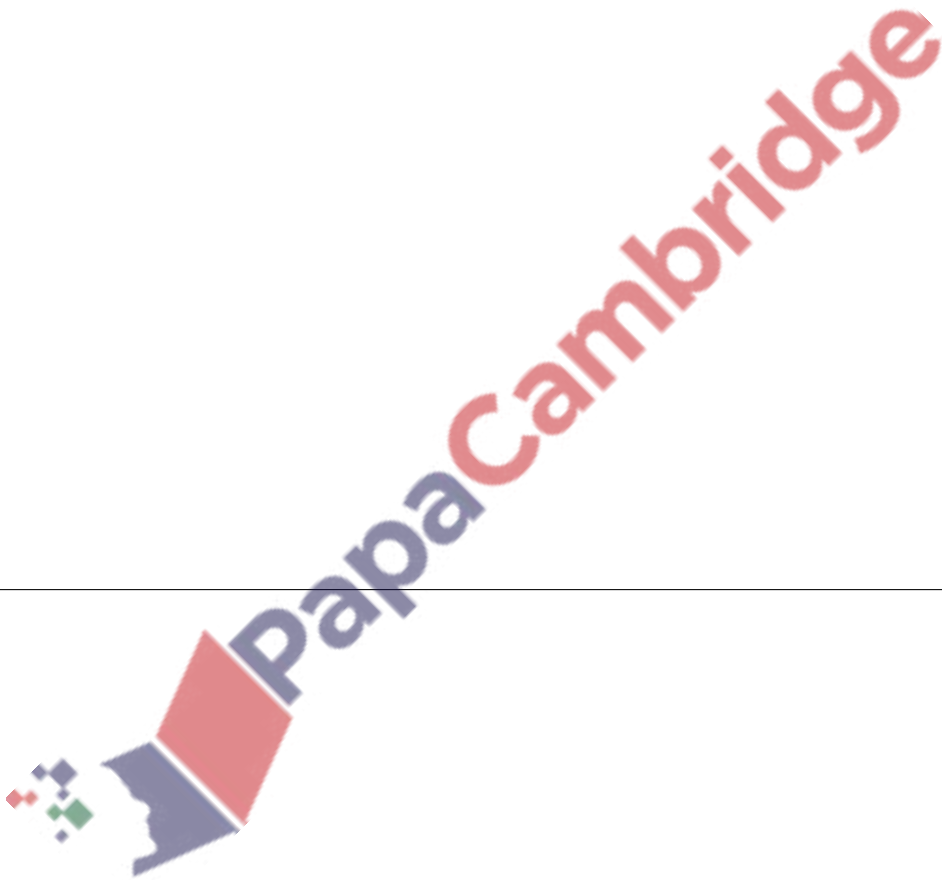
at the point $(1, 1)$. [4]

(b) The parametric equations of a curve are

$$x = \frac{10}{t} - t, \qquad y = \sqrt{(2t - 1)}.$$

Find the gradient of the curve at the point $(-3, 3)$. [6]

75. 9709_w15_qp_21 Q: 1

Use logarithms to solve the equation

$$5^{x+3} = 7^{x-1},$$

giving the answer correct to 3 significant figures.   [4]

76. 9709_w15_qp_22 Q: 1

   (i) Solve the equation $|3x - 2| = 5$.   [3]

   (ii) Hence, using logarithms, solve the equation $|3 \times 5^y - 2| = 5$, giving the answer correct to 3 significant figures.   [2]

77. 9709_w15_qp_22 Q: 3

ln *y*

(0.22, 3.96)

(1.32, 2.43)

O

ln *x*
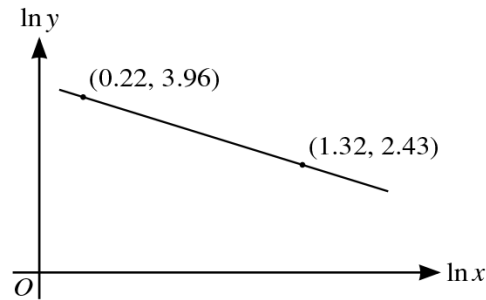
The variables *x* and *y* satisfy the equation $y = Kx^m$, where *K* and *m* are constants. The graph of ln *y* against ln *x* is a straight line passing through the points (0.22, 3.96) and (1.32, 2.43), as shown in the diagram. Find the values of *K* and *m* correct to 2 significant figures. [6]
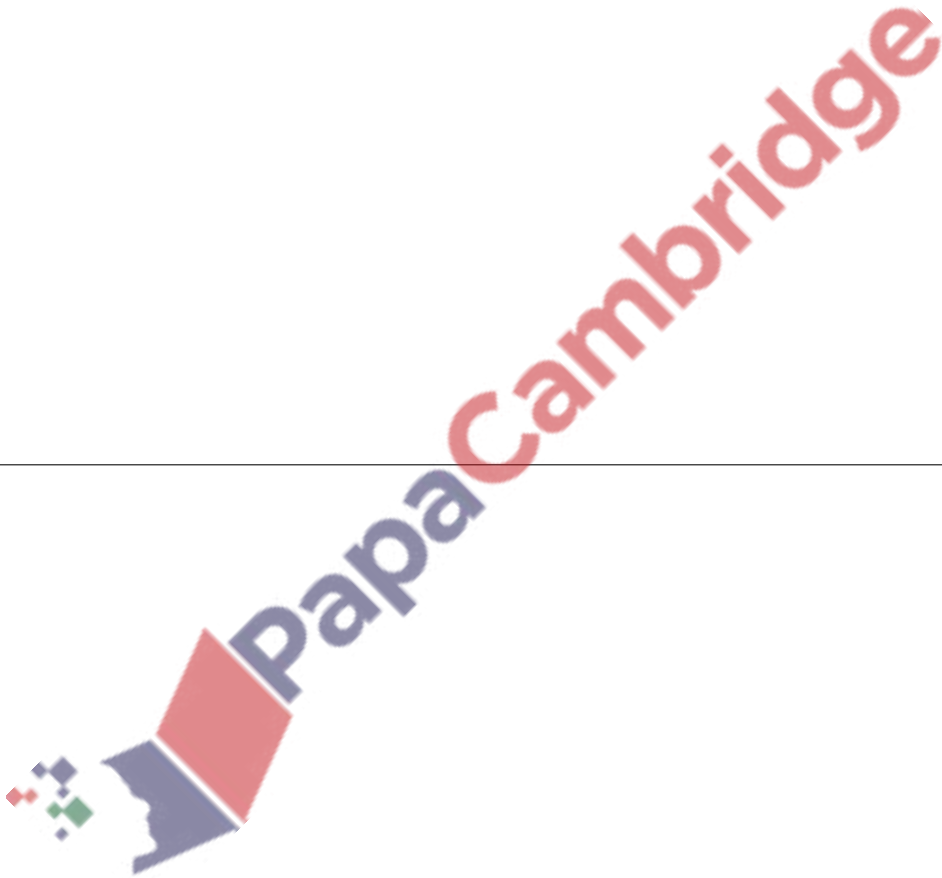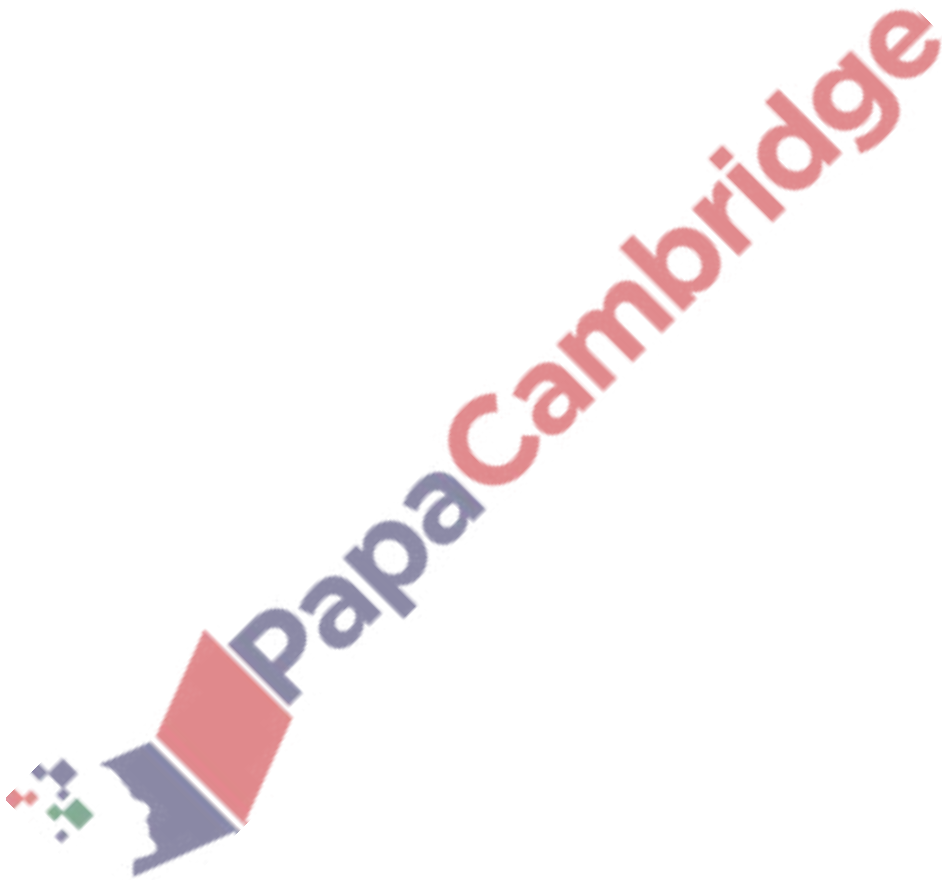
78. 9709_w15_qp_23   Q: 2

(i) Solve the equation $|2x + 3| = |x + 8|$. [3]

(ii) Hence, using logarithms, solve the equation $|2^{y+1} + 3| = |2^y + 8|$. Give the answer correct to 3 significant figures. [2]